ELSEVIER European Economic Review 41 (1997) 559–569

EUROPEAN ECONOMIC REVIEW

Is the natural rate a reference point?

Marika Karanassou *, Dennis J. Snower

Department of Economics, Birkbeck College, University of London, 7 Gresse Street, London W1P 1PA, UK

Abstract

This paper explores the two common concepts of the natural rate of unemployment: (i) the stable, long-run equilibrium rate of unemployment, and (ii) the equilibrium unemployment rate at which there is no tendency for this rate to change, given the exogenous variables. The first concept (common in the theoretical literature) is impractical for empirical assessment, since it requires reliable estimates of the long-run values of the exogenous variables (which are not available) and since it is inherently unable to provide an analysis of how the NRU changes through time. Consequently the second concept is used in empirical studies. The paper shows that this latter natural rate is not necessarily a reference point (a value toward which the equilibrium unemployment rate tends with the passage of time). Specifically, it is not a reference point in multi-equation labor market models containing lagged endogenous variables and exogenous variables with nonzero long-run growth rates. Since these features are exceedingly common, our analysis casts serious doubts on the usefulness of the natural rate hypothesis as a predictive tool. © 1997 Elsevier Science B.V.

JEL classification: J32; J60; J64; E30; E37

Keywords: Unemployment; Natural rate hypothesis; Labor markets; Employment; Capital accumulation; Population growth; Adjustment costs

* Corresponding author. Tel.: (+44 171) 631.6430; e-mail: MKaranassou@Economics.BBK.AC.UK. Snower: tel: (+44) 171 631.6408; e-mail: DSnower@Economics.BBK.AC.UK

PII S0014-2921(97)00022-6

1. Introduction

There is broad agreement among economists that unemployment tends towards the 'natural rate of unemployment' (NRU), i.e. the NRU is a point of reference to which the actual unemployment rate moves. This feature is a major reason why the natural rate hypothesis has become so influential. Economists admit that the NRU is a theoretical construct that is hard to define comprehensively and even more difficult to measure; but if it truly is the rate toward which the unemployment rate tends, then it is not surprising that so much effort should have been lavished on specifying and evaluating it.

This paper takes a fresh look at this approach. For the purpose of our analysis, we will define the 'reference point' y^r of a time-series y_t as the value toward which y_t tends with the passage of time:

$$y^r = \lim_{t \to \infty} y_t. \tag{1}$$

We will examine the conditions under which the NRU is a reference point of the equilibrium unemployment rate (to be defined below).

Investigating whether the natural rate is a reference point is particularly important in the context of European labor markets, because it is commonly argued that the secular rise in European unemployment over the past two decades is due to an increase in Europe's underlying natural rate. Most economists consider it manifestly implausible to suppose that, on account of persistent expectational errors, long-term intertemporal substitution, or other adjustment dynamics, European unemployment could have diverged progressively from its natural rate over this period. Consequently, it is alleged, the key to understanding Europe's unemployment problem must be to find an explanation for the inexorable rise in Europe's natural rate reference point. This accounts for the prodigious econometric efforts to estimate the NRU for European countries.

The aim of this paper is to call into question this pervasive conventional wisdom, that the natural rate – as conventionally defined for empirical purposes – is a reference point. In Section 2 we consider alternative definitions of the NRU and present the standard model where the NRU, thus defined, is a reference point. Section 3 analyzes the circumstances when unemployment does not tend towards its natural rate. The reason why this feature has been overlooked thus far, as we shall see, is that the NRU is usually derived in the context of a single equation, namely, one in which the actual unemployment rate is equal to the natural rate ('structural unemployment') plus a term representing cyclical variations ('cyclical unemployment'). This equation is generally understood as a reduced-form summary of an underlying multi-equation system that includes labor demand and supply equations. But once the natural rate is analyzed in terms of such a multi-equation system, the reference point property of the natural rate can no longer be taken for granted. Rather, as we shall see, the unemployment predicted by such a system may differ systematically from the system's standard, empirically defined natural rate, even if this natural rate is stationary through time.

2. The natural rate as a reference point

The literature on the natural rate hypothesis provides various concepts of the NRU, all of which have three features in common: (i) the NRU is unique, (ii) it depends only on real (not monetary) variables, and (iii) it is the rate at which there are no errors in wage–price expectations. As such, the natural rate hypothesis challenged the traditional Keynesian view that there is a continuum of unemployment equilibria, each associated with a different level of aggregate demand, and that these equilibria depend, in part, on monetary factors. Beyond these features, the literature provides two main alternative characterizations of the NRU:

Definition (a). The NRU is the stable, long-run equilibrium rate of unemployment.

Definition (b). It is the equilibrium unemployment rate at which there is no tendency for this rate to change at any time t, given the values of the exogenous variables at that time.

The first characterization is common in theoretical work, whereas the second predominates in empirical studies.

In the light of these two views, consider the following conventional model of the natural rate:

$$u_t = au_{t-1} + bx_t + \epsilon_t, \tag{2}$$

where a is a scalar ($|a| < 1$ so that the equation is dynamically stable), x_t is a vector of exogenous variables, b is the associated row vector of coefficients, and the error term $\epsilon_t \sim \text{i.i.d.}(0, \sigma^2)$ may be interpreted as errors in wage–price expectations. In what follows, we will take 'equilibrium' to mean 'the absence of expectational errors', so that $\epsilon_t = 0$ at the equilibrium unemployment rate u_t^e. To fix ideas, suppose that the linear combination of exogenous variables x_t tends towards a constant θ with the passage of time: $\lim_{t\to\infty} bx_t = \theta$.

The model (2) of the natural rate hypothesis, containing exogenous variables x_t that change through time, is appropriate for analyzing European labor markets, since changes in these exogenous variables may be used to explain the rise in European unemployment.

In this context, Definitions (a) and (b) may each identify a different natural rate. By Definition (a), the NRU is the long-run equilibrium stationary state:

$$u^* = \frac{\theta}{1-a}. \tag{3a}$$

To derive the NRU according to Definition (b), it is convenient to write the equilibrium unemployment rate (u_t^e) as

$$u_t^e = \frac{bx_t}{1-a} - \frac{a}{1-a} \Delta u_t^e \tag{2'}$$

(by Eq. (2)). Then the equilibrium unemployment rate (in period t) at which there is no tendency for this rate to change in period t (i.e. $\Delta u_t^e = 0$), given the values of the exogenous variables (bx_t and a) at time t, is

$$\tilde{u}_t = \frac{bx_t}{1-a}. \tag{3b}$$

Interpreting the lagged unemployment term in Eq. (2) as the result of dynamic adjustment costs, this NRU of Definition (b) may be understood as what the equilibrium unemployment rate would be once these adjustment dynamics have worked themselves out. [1]

Clearly, as long as the exogenous factor bx_t is not at its long-run stationary level θ, the two definitions of the NRU, u^* and $\tilde{u}_t$, do not coincide.

It is easy to see why empirical studies of the natural rate (e.g. Phelps (1994)) adopt Definition (b) rather than Definition (a). *First*, it is generally impossible to find reliable estimates of the long-run value θ of the linear combination of exogenous variables, whereas it is comparatively easy to estimate the current value bx_t of this linear combination. *Second*, even if estimates of the long-run exogenous variables could be found, the associated estimate of the natural rate would often not be useful – particularly in analyzing the rise of European unemployment rates over the past two decades – for when the long-run values of the exogenous variables are not close to their current values, [2] u^* is a poor predictor of the current unemployment rate. *Third*, under Definition (a), each country has a constant natural rate corresponding to the long-run values of the exogenous variables, and thus this definition is inherently unable to provide an analysis of how the NRU changes through time. This issue has been especially important in the study of European unemployment, and only Definition (b) allows one to address it.

Thus it is the NRU of Definition (b) that generates the available annual

[1] For example, in the presence of hiring and firing costs, current employment depends on past employment, since these costs discourage hiring and firing. Then, since unemployment is the difference between the labor force and employment, current unemployment will depend on past unemployment. Such unemployment persistence can also be generated by costs wage adjustment (such as insider membership effects) and labor force adjustment (such as the search costs of entry and exit from the labor force).

[2] For example, if we assume that the European bx_t in the mid-1990s is close to its long-run value, then the bx_t in the mid-1960s must have been far from this long-run value.

estimates [3] of the natural rate for the US, European countries, and elsewhere, and it is this definition that is the focus of our analysis below. This NRU is a reference point when the following condition is fulfilled:

Condition C1. If the NRU were constant at any particular value u', then the equilibrium unemployment rate would tend toward this NRU: $u' = \lim_{t\to\infty} u^{\mathrm{e}}_t$.

In this sense, the natural rate may be viewed as a moving target: the equilibrium unemployment rate would eventually reach the target if the target would stop moving.

For the single-equation model (2), it turns out that the NRU – in the sense of the empirically implementable Definition (b) – is indeed a reference point. To set the stage for our subsequent analysis, it is important to see precisely why this is so. [4]

Recalling that the natural rate $\tilde{u}_t = bx_t/(1-a)$ is the equilibrium unemployment rate that would occur if the adjustment dynamics worked themselves out in each time period, we ask: if this natural rate were constant at some value u', would the equilibrium unemployment rate converge to it? Observe that the only way for the natural rate $\tilde{u}_t$ to be constant, is for bx_t to be constant (by Eq. (3b)). Accordingly, let us suppose that, from some specific time T onwards, the exogenous factor bx_t remains constant at some level θ', associated with the natural rate $u' = \theta'/(1-a)$. Does the equilibrium unemployment rate u^{e}_t approach the natural rate u' as time t goes from T to infinity?

The first step is to observe that the equilibrium unemployment rate in period $T+t$ (where $t \geq 0$) is

$$u^{\mathrm{e}}_{T+t} = \frac{b}{1-a}x_{T+t} - \frac{a}{1-a}\Delta u^{\mathrm{e}}_{T+t} = \frac{\theta'}{1-a} - \frac{a}{1-a}\Delta u^{\mathrm{e}}_{T+t} \qquad (2'')$$

(by Eq. (2′)). Thus the natural rate is a reference point if and only if $\Delta u^{\mathrm{e}}_{T+t} \to 0$ as $t \to \infty$. To show this, we take the mathematical expectation and first difference of Eq. (2) to obtain $\Delta u^{\mathrm{e}}_{T+t} = a\Delta u^{\mathrm{e}}_{T+t-1} + b\Delta x_{T+t}$. The solution of this equation is

$$\Delta u^{\mathrm{e}}_{T+t} = a^t \Delta u^{\mathrm{e}}_T + b\sum_{j=0}^{t-1} a^j \Delta x_{T+t-j}. \qquad (4)$$

[3] We mean the respectable estimates, not the rule of thumb whereby the natural rate is calculated as just an average of past unemployment rates. Since the latter has no theoretical justification, there is no basis for arguing that they are reference points.

[4] Although what follows is not the simplest way of proving that the natural rate is a reference point, we present this proof because it will provide a convenient way of assessing whether the natural rate is a reference point in the more complicated model of Section 3.

Substituting Eq. (4) into Eq. (2″),

$$u^{e}_{T+t} = \frac{\theta'}{1-a} - \frac{a}{1-a}\left(a^t \Delta u^{e}_{T} + b \sum_{j=0}^{t-1} a^j \Delta x_{T+t-j} \right). \quad (5)$$

Now, since the exogenous factor bx_t remains constant at $bx'\Theta'$ from period T onwards, observe that $\sum_{j=0}^{t-1} a^j b \Delta x_{T+t-j} = 0$ in Eq. (5). Furthermore, observe that $\lim_{t\to\infty} a^t \Delta u^{e}_{T} = 0$. Consequently, as time t approaches infinity, the equilibrium unemployment rate approaches the natural rate: $\lim_{t\to\infty} u^{e}_{T+t} = \theta'/(1-a)$.

This analysis indicates that when the natural rate hypothesis is described in terms of a single, linear, dynamically stable Eq. (2), the associated natural rate $\tilde{u}_t$ is a reference point. There is, however, widespread agreement among labor economists that such single-equation representations can provide only limited insight into the sources of unemployment, since they do not make explicit the labor supply and demand behavior from which the unemployment arises. For this reason it has become common to analyze unemployment in the following general terms: Equilibrium wages and employment are determined by the intersection between a labor demand curve and a wage setting curve (in real wage–employment space), and equilibrium unemployment is then represented as the difference between the labor supply (at the equilibrium real wage) and equilibrium employment. This portrayal of labor market activity is consistent with most of the prevailing theories of unemployment (such as the efficiency wage, insider–outsider, or union theories). [5]

We will now show that when unemployment is portrayed along these lines, the natural rate $\tilde{u}_t$ may cease to be a reference point.

3. When the natural rate is not a reference point

3.1. A simple model

To make our argument as transparent as possible, consider a very simple multi-equation model of unemployment determination, in which the only lag that arises is due to employment adjustment costs. [6] Specifically, let λ and κ be the labor and capital employed by a firm and let the firm's profit be $\pi = f(\lambda, \kappa) - g(\lambda, \lambda_{-1})$, where $g(\lambda, \lambda_{-1})$ is a training cost and $f(\lambda, \kappa)$ is the gross profit (not including the training cost). For simplicity, let $f(\lambda, \kappa) = \lambda^A \kappa^B$, where A and B are constants $(0 < A, B < 1,\ 0 < A + B < 1)$; and let the training cost [7] be

[5] See, for example, Layard et al. (1991), Lindbeck and Snower (1989), and Shapiro and Stiglitz (1984).

[6] Extending this model to include other lags does not affect our overall conclusions.

[7] This function may be interpreted in terms of the current employees receiving ongoing training from last period's employees, where the training costs per person are $D(\lambda/\lambda_{-1})^C$.

$g(\lambda, \lambda_{-1}) = D\lambda - 1(\lambda/\lambda - 1)^C$, where C and D are constants ($C \geq 1$, $D > 0$). Note that when $C = 1$, the training cost reduces to $D\lambda$, so that the firm faces no costs of employment adjustment.

Then employment will be set so that the marginal profit is zero: $\partial\pi/\partial\lambda = A\lambda^{A-1}\kappa^B - CD(\lambda/\lambda_{-1})^{C-1}$, which may be written as $\hat{\lambda} = e_0 - e_1\Delta\hat{\lambda} + e_2\hat{\kappa} = 0$, where $\hat{\lambda}$ and $\hat{\kappa}$ are the logs of λ and κ, respectively, and e_0, e_1, and e_2 are nonnegative constants: $e_0 = (\log A - \log C - \log D)/(1 - A)$, $e_1 = (C-1)/(1 - A)$, and $e_2 = B/(1 - A)$. Observe that when there are no employment adjustment costs ($C = 1$), employment depends solely on the capital stock (since $e_1 = 0$), not on past employment. Supposing that the economy contains a fixed number of identical firms (F), the macroeconomic counterpart of the employment equation above is [8] $N_t = \alpha - \beta\Delta N_t + \gamma K_t$, so that

$$N_t = \frac{\alpha}{1+\beta} + \frac{\beta}{1+\beta}N_{t-1} + \frac{\gamma}{1+\beta}K_t, \tag{6a}$$

where N_t is aggregate employment, K_t is the aggregate capital stock, and all variables – except unemployment rate (below) – are in logs. ($\alpha = e_0 - \log F$, $\beta = e_1$, and $\gamma = e_2$.) In the absence of employment adjustment costs, $\beta = 0$; otherwise β is positive.

Let the labor supply (L_t) depend linearly on the population (P_t):

$$L_t = \delta P_t, \tag{6b}$$

where δ is a constant ($0 < \delta < 1$). Then the unemployment rate may be approximated by [9]

$$u_t = L_t - N_t. \tag{6c}$$

We assume, plausibly, that the growth rates of the capital stock (ΔK_t) and of the population (ΔP_t) follow some (unspecified) stationary processes, $\Delta K_t \sim I(0)$ and $\Delta P_t \sim I(0)$, so that K_t and P_t have non-zero long-run growth rates.

In this context, we derive the NRU of Definition (b) along the lines discussed above. Given the values of the exogenous variables K_t and P_t, the unemployment rate has no tendency to change ($\Delta u_t = 0$) only when the labor force L_t and employment N_t have no tendency to change ($\Delta L_t = \Delta N_t = 0$), by Eq. (6c). Now observe that, for a given population P_t, the labor force is invariably constant (by Eq. (6b)); and for a given capital stock K_t, employment has a tendency to change only when the employment adjustment dynamics have not worked themselves out

[8] Just as the gross profit function $f(\lambda, \kappa)$ implicitly contains the wage as a function of the firm's endogenous and exogenous variables, so the macroeconomic Eq. (6a) may be interpreted as the intersection between a conventional macro labor demand curve and wage setting curve (so that the wage has been 'substituted out' of the employment equation).

[9] In other words, the unemployment rate (the level of unemployment as proportion of the labor force) is approximately equal to the log of the labor force minus the log of employment.

(by Eq. (6a)). Thus the NRU interpreted as the rate that would occur if employment adjusted fully to the capital stock in each time period, so that the employment equation would be $N_t^{\mathrm{n}} = \alpha + \gamma K_t$. In other words, the NRU is

$$u_t^{\mathrm{n}} = L_t - N_t^{\mathrm{n}} = \delta P_t - \alpha - \gamma K_t. \tag{6a$'$}$$

Recall that, by Condition C1, this natural rate is a reference point provided that, if this NRU were constant ($\delta P_t - \alpha - \gamma K_t = u^{\mathrm{n}}$), then the unemployment rate u_t would converge to it ($\lim_{t \to \infty} u_t = u^{\mathrm{n}}$). To investigate whether this is the case, we follow the same procedure as in the previous section: we assume that, from period T onwards, the population and the capital stock evolve in such a way that $\delta \Delta P_t = \gamma \Delta K_t$, so that the NRU is constant: $u_t^{\mathrm{n}} = u^{\mathrm{n}}$. To find whether the unemployment rate approaches u^{n} with the passage of time, we first observe that the unemployment rate in period $T + t$ is

$$u_{T+t} = \delta P_{T+t} - \alpha + \beta \Delta N_{T+t} - \gamma K_t = u^{\mathrm{n}} + \beta \Delta N_{T+t} \tag{7}$$

(by Eqs. (6a), (6b) and (6c)). To derive ΔN_{T+t}, we take the first difference of Eq. (6a) to obtain $\Delta N_{T+t} = (\beta/(1+\beta)) \Delta N_{T+t-1} + (\gamma/(1+\beta)) \Delta K_{T+t}$. The solution of this equation is

$$\Delta N_{T+t} = \left(\frac{\beta}{1+\beta} \right)^t \Delta N_T + \left(\frac{\gamma}{1+\beta} \right) \sum_{j=0}^{t-1} \left(\frac{\beta}{1+\beta} \right)^j \Delta K_{T+t-j}. \tag{8}$$

Substituting (8) into (7),

$$u_{T+t} = u^{\mathrm{n}} + \beta \left(\left(\frac{\beta}{1+\beta} \right)^t \Delta N_T + \left(\frac{\gamma}{1+\beta} \right) \sum_{j=0}^{t-1} \left(\frac{\beta}{1+\beta} \right)^j \Delta K_{T+t-j} \right). \tag{9}$$

Observe that $\lim_{t \to \infty} (\beta/(1+\beta))^t \Delta N_T = 0$ and thus, by Eq. (9), we find that

$$\lim_{t \to \infty} u_{T+t} = u^{\mathrm{n}} + \beta \left(\left(\frac{\gamma}{1+\beta} \right) \sum_{j=0}^{\infty} \left(\frac{\beta}{1+\beta} \right)^j \Delta K_{T+t-j} \right). \tag{10}$$

Here the unemployment rate does *not* approach the natural rate with the passage of time since the capital stock has a nonzero long-run growth rate, and thus $\sum_{j=0}^{t-1} (\beta/(1+\beta))^j \Delta K_{T+t-j}$ does not approach zero as time approaches infinity. The NRU is no longer a reference point.

In order to assess the discrepancy between the reference point and the NRU, we would need to know the magnitudes of the growth rates of the capital stock from time $T+1$ into the infinite future. In practice, of course, this is obviously impossible. In short, the empirically implementable measure of the natural rate (u^{n}) is of little relevance in predicting the movement of unemployment, since it is not a reference point and since the discrepancy between this natural rate and the reference point is not empirically assessable.

3.2. The intuition

The intuitive reason why the unemployment rate does not tend towards the NRU is straightforward. Recall that since the NRU is the unemployment rate at which there is no tendency for this rate to change in any time period t, given the exogenous variables K_t and P_t, it thereby represents the unemployment rate that would prevail if the process of employment adjustment worked itself out in each period t. However, *in the labor market equilibrium, this process never works itself out completely*. The reason, of course, is that the capital stock is continually increasing, and thus employment is continually rising in response. Since the labor supply is rising in tandem with the capital stock, the NRU remains constant through time. But the equilibrium unemployment rate never approaches the natural rate, since employment never catches up with its moving target, set by the rising capital stock.

As we have seen, this cannot happen in a dynamically stable, single-equation model of unemployment, and it should now be clear why. For if the natural rate were constant in this model, then the equilibrium unemployment rate would tend towards this natural rate in the long run, simply as consequence of dynamic stability. In other words, a single-equation model can only portray adjustment dynamics that *do* work themselves out in the long run.

But in a multi-equation model of unemployment, the natural rate need not be a reference point, and the analysis above indicates that this occurs when the model contains:

(i) lagged endogenous variables (such as lagged employment in the Eq. (6a)) and

(ii) non-stationary exogenous variables with nonzero long-run growth rates (such as the capital stock in Eq. (6a) and the population in Eq. (6b)). [10]

These features are so common in labor market models containing labor demand and supply functions, as to be virtually ubiquitous. Thus, the phenomenon above – a natural rate toward which the equilibrium unemployment rate does not tend – is not a mere theoretical curiosity. Karanassou and Snower (1996) present an econometric model that shows this to be an empirically important matter.

4. Summary and conclusions

In sum, we have argued that the concept of the NRU as a stable long-run equilibrium unemployment rate (Definition (a)) is impractical and unhelpful for

[10] Karanassou and Snower (1996) show, however, that the natural rate can be a reference point even if these two conditions hold, provided that both the labor demand and labor supply equations contain autoregressive coefficients and that these coefficients are identical. But this could happen only by accident.

empirical assessment, and thus empirical estimates of the natural rate are based on another concept – the equilibrium rate at which there is no tendency for this rate to change, given the values of the exogenous variables (Definition (b)). Our analysis showed that this latter natural rate is *not* a reference point in multi-equation labor market models containing lagged endogenous variables as well as exogenous variables with nonzero growth rates. Since lagged endogenous variables are a common feature of labor market models due to the pervasiveness of labor market adjustment costs, and since non-stationary exogenous variables are common on account of economic growth, we argued that this issue is likely to be of considerable practical importance.

These results cast doubt on the usefulness of the natural rate hypothesis as a predictive tool; for if the equilibrium unemployment does not tend towards the natural rate with the passage of time, it is no longer clear why this natural rate should claim our attention.

There is of course a way out of this box, and that is to estimate multi-equation models of the labor market and to use these to predict the unemployment rate. In such models the exogenous variables typically keep moving through time and the unemployment effect of these movements is typically amplified through the interactions of the lags in the constituent equations. Since the exogenous variables generally do not remain constant long enough for the adjustment dynamics to work themselves out, the predicted unemployment rates in any time period usually differ substantially from the long-run stationary state (the unemployment reference point) in that time period. [11] Consequently, this approach is far removed from the standard single-equation specifications of the natural rate hypothesis.

Acknowledgements

We are very grateful to Ron Smith and Gylfi Zoega for insightful comments.

References

Henry, Brian and Dennis J. Snower, eds., 1996, Economic policies and unemployment dynamics in Europe (IMF, Washington, DC) forthcoming.

Karanassou, Marika and Dennis J. Snower, 1993, Explaining the disparities in European unemployment dynamics, Rivista di Politica Economica 11, 37–62.

Karanassou, Marika and Dennis J. Snower, 1995, A contribution to unemployment dynamics, Discussion paper no. 1176 (CEPR, London).

Karanassou, Marika and Dennis J. Snower, 1996, A critique of the natural rate hypothesis, Mimeo.

Layard, Richard, Stephen Nickell and Richard Jackman, 1991, Unemployment: Macroeconomic performance and the labour market (Oxford University Press, Oxford).

[11] See, for example, Karanassou and Snower (1993, 1995) and Henry and Snower (1996).

Lindbeck, Assar and Dennis J. Snower, 1989, The insider–outsider theory of employment and unemployment (MIT Press, Cambridge, MA).

Phelps, Edmund S., 1994, Structural slumps: The modern equilibrium theory of unemployment, interest and assets (Harvard University Press, Cambridge, MA).

Shapiro, Carl and Joseph E. Stiglitz, 1984, Equilibrium unemployment as a worker discipline device, American Economic Review 74, no. 3, 433–444.